Applied Physics Lecture Material
Unit-I
QUANTUM MECHANICS
for
B.Tech (ECE, AI&ML) I year – I semester
R18 Regulation

Unit-I Objectives:

1. Student will demonstrate skills in scientific inquiry, problem solving and laboratory techniques.

2. Student will able to demonstrate competency and understanding the concepts found in quantum mechanics.

Unit-I Outcomes:

1. The students will able to learn the fundamental concepts on quantum behaviour of matter in its micro state.

2. The students will able to apply these concepts for semiconductor physics.

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UNIT-I

QUANTUM MECHANICS

Introduction to Quantum Physics

- In physics Classical mechanics and Quantum mechanics are the two major sub fields of Mechanics.
- Classical mechanics describing the motion of macroscopic particles.
- Classical mechanics fails to explain the motion of atomic particles such as electrons, protons etc. The failures of classical mechanics are
    1. Black body radiation
    2. Photo electric effect
    3. Compton effect
    4. Atomic Spectra
- Quantum mechanics began in 1900 when the study of light emitted by heating solids was studied, so we begin by discussing *the nature of light*.
- In 1801Thomas Young gave convincing experimental evidence for the wave nature of light by showing that light exhibited diffraction and interference when passed through two adjacent pinholes.
- In 1860 Maxwell developed Maxwell's equations predicted that an accelerated electric charge would radiated energy in the form of electromagnetic waves.
- Electromagnetic (EM) radiation travels through space as electric energy and magnetic energy.

  Wavelength λ and frequency ν are related by:

$$c = \nu\lambda = 3 \times 10^8 \text{ m/s}$$

- Frequency (or wavelength) determines the type of radiation. All electromagnetic waves travel at speed $c = 3\times10^{10}$ cm/sec in vacuum.
- The light is a form of energy and it exhibits particle and wave properties (i.e., dual nature)
- As a wave, we can describe the energy by its wavelength, which is the distance between two successive crests or troughs.
- The wavelength of light is more commonly stated in nanometres (nm). One nanometre is one billionth of a meter.
- Visible light has wavelengths of roughly 400 nm to roughly 780 nm. This range of wavelengths is called the visible spectrum.
- In 1900 Max Planck proposed quantum hypothesis. It states that the emission or absorption of electromagnetic radiations takes place in the form of quanta or photons, which are having discrete energies only.

- Each quanta or photon has energy **E=hυ,** where **h** is the Plank's constant and **υ** is the frequency of radiation. These concepts led to a new mechanics which is known as quantum mechanics.

1.What is meant by quantum physics.

Ans.

- Quantum physics is a branch of science that deals with discrete indivisible units of energy is called quanta. There are five main ideas represents in the Quantum theory
    1. Energy not continues but comes in small discrete units
    2. The elementary particles behave both like particles as well as waves.
    3. The moment of these particles inherently random.
    4. It is physically impossible to know both the position and momentum of a particle at the same time.
    5. The atomic word is not like the word we live in.

2. What is quantum theory of light.

Ans.

- Quantum theory of light proposed by Max Plank presents particle picture of light. Light energy is traveling in the form of a tiny packets is called as quanta.
- The energy of each quanta is **E=h*ν***

    where **h** is planks constant and **υ** is frequency of radiation
- This theory successfully explains energy distribution in black body radiation, photo electric effect and Compton effect.

3. What is black body radiation.

Ans.

- A perfect black body absorbs radiation of all wavelength's incident on it. It also emits radiation of all wavelengths
- When a black body is at higher temperature then its surroundings, the radiation emitted is called black body radiation. The radiation emitted varies with temperature.

4. What is meant by energy spectrum of a black body.

Ans

- When a black body is at higher temperature than its surrounding, the radiation emitted is called black body radiation. The radiation emitted varies with temperature.
- The intensity of radiation corresponding to different wavelengths is measured at different temperatures and plotted as energy spectrum of a black body.

5. State Wiens displacement law of black body radiation.

Ans.

- According to Wiens displacement law, in the energy spectrum of a black body, the product of the wavelength corresponding to maximum energy(λ_m) and absolute temperature is constant.

  i.e.,$\lambda_m T$ = Constant.

6. What is Rayleigh-Jeans law, of radiation.

Ans.

- According to Rayleigh-Jeans law, the energy distribution in the black body spectrum is given by

$$E_\lambda = 8\pi kT/\lambda^4$$

7.Deduce Rayleigh-Jeans law from Planks of radiation.

Ans.

- When λ is very large, then $\exp\left(\frac{hc}{\lambda KT}\right) \approx 1 + \frac{hc}{\lambda KT}$. From Planck's formula, we have

$$E_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{1+\left(\frac{hc}{\lambda KT}\right)-1}$$

$$E_\lambda\, d\lambda = \frac{8\pi KT}{\lambda^4}\, d\lambda$$

This is Rayleigh-Jeans Law.

8. State Rayleigh jean's law. What are its limitations.

Ans.

- According to Rayleigh jean's law the energy distribution in the thermal spectrum is given by

$$E_\lambda = 8\pi kT/\lambda^4$$

Where **K** is the Boltzmann's constant

- The Rayleigh jean's law holds good in the region of longer wavelength but not for shorter wavelength.

9. State planks law of black radiation (or) write planks radiation formula.

Ans.

- According to planks theory energy is emitted in the form of packets or Quanta called photons and energy of photons is given by **E=nh*ν*** where **n**=1,2,3 etc.
- In black body, total energy of photons with the wave length range $\lambda$and $\lambda + d\lambda$ is given by

$$E_\lambda d\lambda = \frac{8\pi hc}{\lambda^5\left(e^{\frac{h\nu}{kT}} - 1\right)} d\lambda$$

Where **h** = Planck's constant, **C**= speed of the light, **T** = temperature of the enclosure

10.What is photoelectric effect.

Ans.

- The phenomenon of the emission of electron from certain metal surfaces when electromagnetic radiation (such as X-rays, ultraviolet rays and in some cases visible light) of sufficient energy falls on them is called **Photoelectric effect**. The electrons ejected are called photoelectrons.

Photoelectric equation:

$$h\nu = \frac{1}{2} mv^2 + \Phi$$

$$K.E. = h\nu - \Phi$$

Where K.E. is the Kinetic energy of ejected electron, **hν** is the Energy of the Incident Photon and Φ is the work function.

Work function: The minimum energy ($h\nu_0$) required for emission of electron from the metal surface is known as work function.

Work function (Φ)= $h\nu_0$

where ν_0 is threshold frequency which depend upon the nature of the metal.

11.What is Compton effect.

Ans.

- Compton found that when a beam of monochromatic X-ray passes through matter, the scattered beam consists of two types of radiation.one is the primary radiation of the same wavelength while the other is a modified radiation of slightly longer wave length.
- Compton wavelength is difference between the scattered photon wavelength **λ'** and incident photon wave length **λ**it can be written as

$$\lambda - \lambda' = \frac{h}{mc}(1-\cos\phi).$$

12. For a free particle moving within a one-dimensional potential box, the ground state energy cannot be zero. Why

Ans.

The three integers n_1, n_2, and n_3 called **quantum numbers** are required to specify completely each energy state. Since for a particle inside the box, ψ cannot be zero, no quantum number can be zero. Hence ground state energy cannot be zero.

13. In detail describe Black Body Radiation.

Ans.

Laws of Radiation:

- All objects emit radiant energy. Hotter objects emit more energy than colder objects (per unit area).

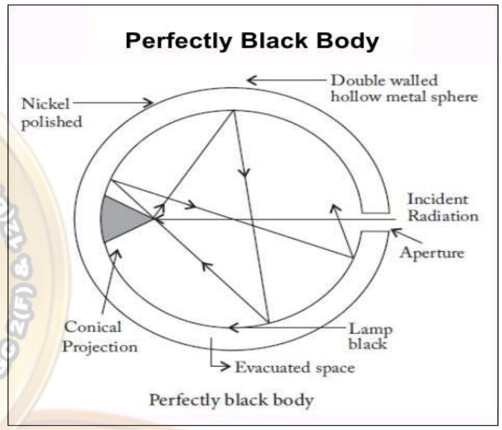
- The amount of energy radiated is proportional to the temperature of the object. The hotter the object, the shorter the wavelength (λ) of emitted energy.
- Hot objects emit electromagnetic radiations over a wide range of wavelengths.
- Intensities of radiations of different wavelengths emitted by hot body depend upon its temperature.

Black body: The ideal body which emit and absorbs all the frequencies of radiation is called Black body.

- The radiation emitted by Black body is called Black body radiation.

Construction of Black body:

- The black body consists of a metallic hollow sphere painted with lamp black internally and provided a small hole to enter the radiation.
- In front of the hole, there is a projection to prevent the direct reflection of radiation from inner surface.
- When radiation enters the hollow space through this hole, it undergoes multiple reflections and finally absorbed.
- When this body is placed in a bath at higher temperature, heat radiation comes out of the hole. The hole acts as a black body radiator.



Figure(a) Black Body

Energy spectrum of a black body

Let us study how energy is distributed in the spectrum of black body. The radiation emitted by a black body varies with temperature. The intensity of radiation corresponding different wavelengths at different temperatures is plotted shown in figure (b)

From this study is observed that

- At a given temperature, the energy is not uniformly distributed in the radiation spectrum.
- At a given temperature, the intensity of radiation is maximum at particular wavelength λ_m.
- With increase in temperature, λ_m decreases.
- For all wavelengths, an increase in temperature causes an increase in the energy emission.

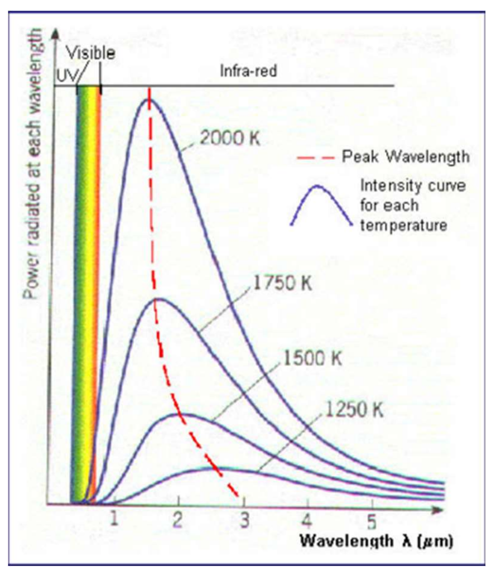


Figure (b) Energy spectrum of black body

14. Derive Planck's Law of radiation.
Ans.

- According to the Planck's quantum theory, a black body is composed of large number of oscillating particles(oscillators) i.e., atoms in the wall of black body, which can vibrate with all possible frequencies.
- According to the Planck's quantum theory of heat radiation.

  (a) The radiation energy emitted or observed in the form of small packets of energy (in discrete quantities and not in a continuous manner). Such packets are known as quantum or photon.

  (b) Energy of each photon is directly proportional to frequency of radiation.

  The energy of photon is

  $E \ \alpha \ h\nu$

  $E = nh\nu$

  Where 'h' is the Planck's constant **$h=6.626\times10^{-34}$Js** and **ν** is the frequency of radiation

- The energy of oscillating particle is quantized.

  i.e., $E=0, h\nu, 2h\nu, 3h\nu$............ $nh\nu$ where n = 0,1,2,3,4……

- Let 'N' be the total number of Planck's oscillators and 'E' be their total energy.

  Then the average energy per oscillator is given by

  $\bar{E} = \frac{E}{N}$ ----------- (1)

- Let $N_o, N_1, N_2,$ -----------N_r be the number of oscillators having energies

  $0, h\nu, 2h\nu, 3h\nu$ ----- $rh\vartheta$ respectively.

  Therefore, the total number of oscillators is

  $N= N_o+ N_1+ N_2+$ .... $+ N_r$ ----------- (2)

  and total energy is given by

  $E = 0 + h\nu + 2h\nu +$.............. $nh\nu$ -------------(3)

According to Maxwell's distribution law, the number of oscillators having energy rE is given by

$N_r=N_0 \exp (-rE /kT)$ ------------- (4)

N_r is the number density of molecules in r^{th} state (i.e., the number of molecules per unit volume). T is the temperature, and k_B is Boltzmann's constant. 1.38×10^{-23} m^2 kg s^{-2} K^{-1}.

Substituting the values of N_1, N_2, N_3…………by substituting r = 1,2,3........ from equation (4) in equation (2) We get

$$N = N_0 + N_0\exp(-\frac{hv}{KT}) + N_0\exp(-\frac{2hv}{KT}) + \ldots\ldots\ldots$$

$$= N_0(1+\exp(-\frac{hv}{KT}) + \exp(-\frac{2hv}{KT}) + \ldots\ldots\ldots\ldots\ldots\ldots$$

$$N = N_0 / (1-\exp(-\frac{hv}{KT}) \quad \text{-------- (5)} \qquad \text{(since } 1+x+x^2+\ldots\ldots=1/1-x)$$

Similarly in equation (3) substituting the values of N1, N2, ... we get

$$E = (0\times N_0) + hv\times N_0\exp(-\frac{hv}{KT}) + 2hv \times N_0\exp(-\frac{2hv}{KT}) + \ldots\ldots..$$

$$E = N_0 hv \exp(-\frac{hv}{KT}) [1+2\exp(-\frac{hv}{KT})+3\exp(-\frac{2hv}{KT})+\ldots\ldots\ldots\ldots]$$

$$E = N_0 \exp(-\frac{hv}{KT})[hv/ \{(1-\exp(-\frac{hv}{KT})\}^2] \quad \text{(Since } 1+2x+3x^2+\ldots\ldots = 1/(1-x)^2)$$

Substituting the values of N and E in equation (1) we get the average energy of the oscillator.

$$\bar{E} = \frac{E}{N}$$

$$\bar{E} = N_0\exp(-\frac{hv}{KT})[hv/ \{(1-\exp(-\frac{hv}{KT})\}^2]/ N_0 / (1-\exp(-\frac{hv}{KT})$$

$$\bar{E} = hv \exp(-\frac{hv}{KT})/(1-\exp(-\frac{hv}{KT})$$

$$\bar{E} = \frac{hv}{\exp\left(\frac{hv}{KT}\right)-1} \quad \text{----------- (6)}$$

This is the expression for average energy of a Planck's oscillator

The no. of oscillators per unit volume in the frequency range of *v* and ***v+dv*** are given by

$$N = \frac{8\pi v^2}{c^3}\, dv \quad \text{----------(7)}$$

The energy density of radiation (E_v) in the frequency range ***v*** and ***v+dv*** of oscillator is given by

$$E_v\, dv = \frac{8\pi v^2}{c^3}\, dv \times \frac{hv}{\exp\left(\frac{hv}{KT}\right)-1}$$

$$E_v\, dv = \frac{8\pi h v^3}{c^3}\frac{dv}{\exp\left(\frac{hv}{KT}\right)-1} \quad \text{---------(8)}$$

This is the Planck's radiation law in terms of frequency.

This can also be expressed in terms of wavelength(λ)

$$E_\lambda\, d\lambda = \frac{8\pi hc}{\lambda^5}\frac{d\lambda}{\exp\left(\frac{hc}{\lambda KT}\right)-1} \quad \text{---------(9)} \qquad (v = \frac{c}{\lambda}\ ,\ dv = \left|-\frac{c}{\lambda^2}\right| d\lambda\ )$$

Different Laws of from Planck's Formula:

i) Wien's Law: When λ is very small, then $\exp\left(\frac{hc}{\lambda KT}\right) \gg 1$. So neglected 1 in the denominator from Planck's formula (9) we have

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{\exp\left(\frac{hc}{\lambda KT}\right)} \quad \text{---------(10)}$$

This is Wien's law.

ii) Rayleigh-Jeans Law: When λ is very large, then $\exp\left(\frac{hc}{\lambda KT}\right) \approx 1 + \frac{hc}{\lambda KT}$. From Planck's formula (9) we have

$$E_\lambda \, d\lambda = \frac{8\pi hc}{\lambda^5} \frac{d\lambda}{1+\left(\frac{hc}{\lambda KT}\right)-1}$$

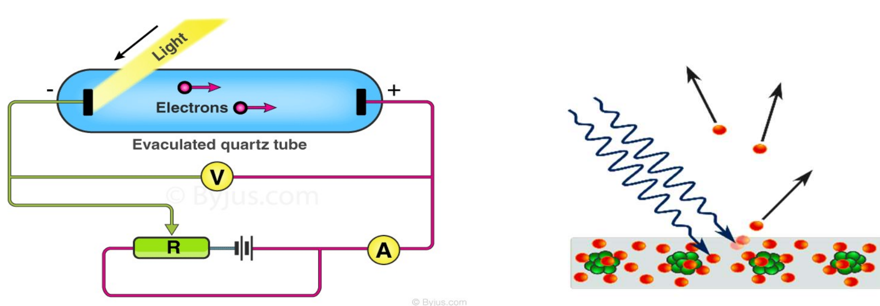
$$E_\lambda \, d\lambda = \frac{8\pi KT}{\lambda^4} \, d\lambda \quad \text{-----------------(11)}$$

This is Rayleigh-Jeans Law.

15. Describe Photo electric effect in detail.

Ans.

- The photo electric effect was discovered in 1887 but was not explained. Einstein in 1905 extended Planks idea and suggested quantum nature of light and quantization of energy in his explanation of the photo electric effect. Einstein received the Nobel prize for physics as his work marked the beginning of quantum theory.
- **Definition:** the phenomenon of the emission of electrons from certain metal surfaces when electromagnetic radiation (such as X-Rays, ultraviolet rays and in some cases visible light) of sufficient energy fall on them is called Photo electric effect. The electrons ejected are called photoelectrons.



Fig(a) schematic of the apparatus for studying the photoelectric effect

Experimental study:

- The photo electric effect consists of two metal (Zinc) plates A and C kept inside an evacuated quartz bulb act as anode and cathode respectively.

- When Ultraviolet rays fall on the cathode **C** (which is connected to the negative terminal of a battery), there is flow of current (as indicated by ammeter).
- But when light falls on the anode **A** (which is connected to the positive terminal of the battery) there is no flow of current.
- From these observations we realise that the particles emitted by the cathode when ultra violet rays fall on it nothing but electrons.
- These emitted electrons are attracted by the anode **A** causing flow of current

.

- In this experiment photo electric current is studied as a function of

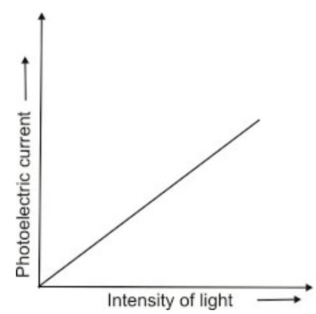
  i) The intensity of incident light

  ii) The frequency of incident light

  iii) The potential difference between anode and cathode.

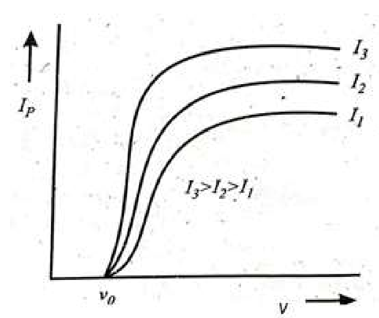
i) Variation of photo electric current with Intensity of incident light

- Keeping the potential difference between anode and cathode as well as frequency of incident light is constant.
- The intensity of incident light increases when the photo electric current increases.



ii) Variation of photo electric current with frequency

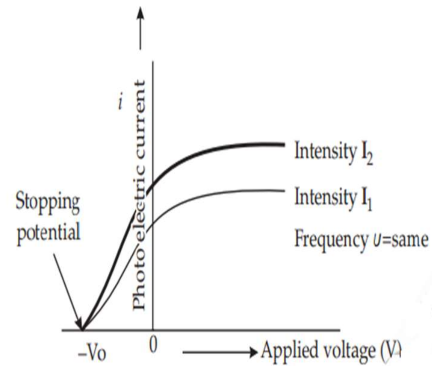
- Keeping the intensity of incident light as well as the potential difference between anode and cathode is constant, the frequency of incident light is increased.
- When the frequency is below a particular value there is no photoelectric current whatever may be the intensity of incident light.
- Thus, the minimum frequency of incident light below which there is no photoelectric current and the minimum frequency is called **threshold frequency(ν_o)**
- Figure (B) shows the variation of photo electric current I_p with frequency for various intensities of incident light I_1 I_2 I_3etc.



Fig(b) Variation of photo electric current with frequency

iii) Variation of photo electric current with applied voltage

- Keeping the intensity of incident light as well as the frequency of incident light is constant, the photo electric current is measured by varying the potential difference between anode and cathode.
- From higher negative voltage the potential is gradually reduced to zero and then increased to positive values.



Fig(c) Variation of photo electric current with applied voltage

- When the potential is below a minimum value is called **stopping potential or cut-off potential ($-V_0$)**, photoelectric current is zero irrespective of intensity of incident light.
- When the potential is increased towards positive, after stopping potential the photo electric current increases rapidly and finally reach a study maximum value is called the saturation current.
- The experiment repeated for different intensities of incident light $I_1\ I_2\ I_3$ etc.as shown in Fig(c).

Einstein's photo electric equation

- Einstein could explain photoelectric effect using Plank's Quantum theory as follows. Photoelectrons are ejected only when incident light has threshold frequency. If frequency of incident light is more than threshold frequency then the excess energy is imparted to electrons in the form of kinetic energy.
- According to Einstein, the energy of incident photon is equal to the summation of work function and kinetic energy of photo electron.

  i.e., $h\nu = W_0 + K.E$ ----------------- (1)

  Where W_0 is the work function
- **Work function**: The minimum energy required to cause the photo-electric effect is known as work function. i.e., $W_0 = h\nu_0$

  Were $\nu_0$is the threshold frequency which is depend upon nature of the metal.
- When the frequency of incident light ν_0 is just the threshold value i.e.,$\nu=\nu_0$ then the electron is just emitted but with zero velocity.
- When $\nu$is greater than $\nu_0$energy $h\nu_0$ is utilised to release the electron.
- The energy ($h\nu$-$h\nu_0$) is appearing as kinetic energy of the electron.
- If **m** is the mass of electron **v** is its velocity, then its K.E is $\frac{1}{2}mv_{max}^2$.

  From equation (1) we have

$$hv\text{-}hv_0=\frac{1}{2}mv_{max}^2\text{------------}(2)$$

This is Einstein's photo electric equation

Laws of photoelectric emission

Law1: The number of photoelectrons emitted per second is proportional to the intensity of incident radiation provided the frequency is above the threshold frequency.

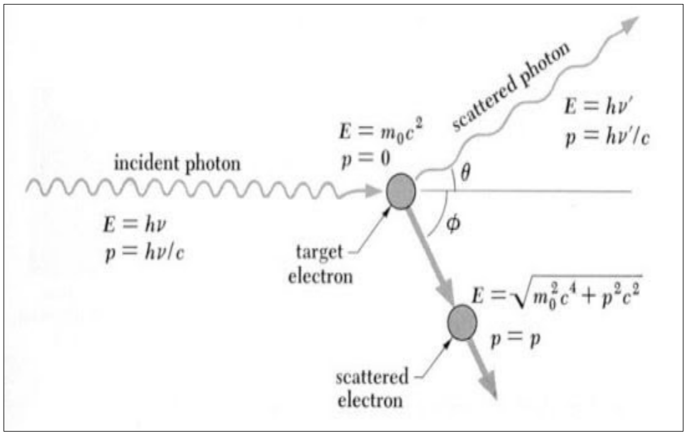
Law2: The photoelectrons are emitted with a range of kinetic energies from zero up to a maximum value. The maximum energy of the emitted photoelectrons increases with the frequency of the incident radiation and is independent of it intensity.

Law3: For a given metal there is certain minimum frequency of radiation, called the ***threshould frequency***,below which no emission occurs irrespective of the intensity of the incident radiation.

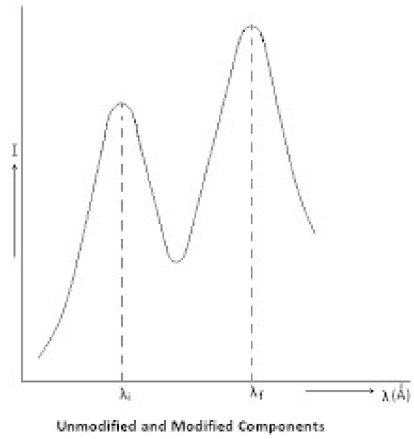
16. What is Compton effect? Derive an expression for thewavelength of scattred x-ray photon from a material.

Ans.

- Compton found that when a beam of monochromatic X-ray passes through matter, the scattered beam consists of two types of radiation, one is the primary radiation of the same wave length while the other is a modified radiation of slightly longer wave length. This effect is called **Compton effect**.
- In 1923 Compton discovered that when a homogeneous beam of X-rays frequency (v) where incident on a light element like Carbon or Aluminum, the X-rays suffered to a change of frequency on Scattering.
- The Scattered beam consists two wavelengths
  - One scattered beam had the same wavelength as the incident beam (λ).
  - The second beam had a wavelength longer than that of primary beam (λ')
- The difference in the wavelength between modified and unmodified different components are called Compton shift ($\lambda' - \lambda = \Delta\lambda$)
- Compton shift is depend on the scattering angle Θ and it is independent of primary wavelength λ and target material
- Compton wave length is difference between the scattered photon wave length λ' and incident photon wave length λ it can be written as $\lambda' - \lambda = \Delta\lambda = \frac{h}{mc}(1\text{-}\cos\Theta)$.



Figure(a) Compton Effect



Figure(b) Intensity Vs Wavelength

Derivation

- Compton assumed that the primary radiation of wavelength λ consists of corpuscles or photons of frequency $\nu=\frac{c}{\lambda}$.
- Let a X-ray photon collide with an electron at rest O , after collision they move in the direction as indicated shown in fig(a).
- Let $h\nu$ *and* $\frac{h\nu}{c}$ be the energy and momentam of incident photon respectively .
- Let $h\nu'$ and $\frac{h\nu'}{c}$ be the energy and momentam of the scattred photon respectively.
- Consider Θ is the scattred angle of photon, Φ is recoil angle of electron and m_0 be the rest mass of electron.
- **Before collision (while electron is at rest)**

Energy of the incident photon= $h\nu$

Momentam of the incident photon= $\frac{h\nu}{c}$

Energy of the electron (at rest)= m_0c^2(according to theory of relativity)

Momentam of the rest electron=**0**

Energy of the system (photon and electron) before collision = $h\nu+m_0c^2$---------(1)

Momentum of the system before collision = $\frac{h\nu}{c}+0$ ------------------------------(2)

After collision (while electron moves with a velocity v)

Energy of the scattered photon= $h\nu'$

Momentum of the scattered photon= $\frac{h\nu'}{c}$

Energy of the recoil electron= mc^2

Momentum of the recoil electron=**mV**

Energy of the system after collision= $h\nu'+mc^2$ ------------------------(3)

Momentum of the system after collision= $\frac{h\nu'}{c}+mV$----------------------------(4)

Laws of consrvation

Applying the principle of conservation of energy (from eqs.1 and 3)

$h\nu+m_0c^2 = h\nu'+mc^2$

i.e., $mc^2 = h(\nu - \nu') + m_0c^2$ ---------------------------(5)

applying the principle of conservation of momentam(from eqs.2 and4)

$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}+mv$----------------------------------(6)

Applying princinpe of conseveration of momentam along the direction of (X --- direction)

$\frac{h\nu}{c} + 0 = \frac{h\nu'}{c}\cos\Theta + mv\cos\Phi$

$mvc\cos\Phi = h\nu - h\nu'\cos\theta$ -------------------------(7)

Applying principle of conseveration of momentam perpenduclar to the direction of incident(Y-Direction)

$0+0 = \frac{h\nu'}{c}\sin\Theta + mv\sin\Phi$

$mvc\sin\Phi = h\nu'\sin\Theta$ ---(8)

squaring and adding eqs (7) and (8)

$m^2v^2c^2[\cos^2\Phi+\sin^2\Phi] = [h\nu - h\nu'\cos\theta]^2+[h\nu'\sin\Theta]$

$m^2v^2c^2 = h^2\nu^2+h^2\nu'^2\cos^2\Theta - 2h\nu h\nu'\cos\Theta + h^2\nu'^2\sin^2\Theta$

$m^2v^2c^2 = h^2[\nu^2+\nu'^2\cos^2\Theta+2\nu\nu'\cos\Theta+\nu'^2\sin^2\Theta]$

$m^2v^2c^2 = h^2[\nu^2+\nu'^2[\cos^2\Theta+\sin^2\Theta]- 2\nu\nu'\cos\Theta]$

$m^2v^2c^2 = h^2[\nu^2+\nu'^2- 2\nu\nu'\cos\Theta]$-----------------------(9)

From eq (5)

$m^2c^4=[h(\nu - \nu')+m_0c^2]^2$

$m^2c^4=[h^2(\nu-\nu')^2+m_0^2c^4+2hm_0c^2(\nu-\nu')]$ ν

$m^2c^4=[h^2(\nu^2+\nu'^2- 2\nu\nu')+m_0^2c^4+2hm_0c^2(\nu-\nu')]$

subtracting eq (10) and (9)

$m^2c^4 - m^2c^2v^2=[h^2(\nu^2+\nu'^2-2\nu\nu')+m_0^2c^4+2hm_0c^2(\nu - \nu')]-h^2[\nu^2+\nu'^2-2\nu\nu'\cos\Theta]$

i.e., $m^2c^2(c^2 - v^2)= m_0c^4+2hm_0c^2(\nu-\nu')]-2h^2\nu\nu'(1- \cos\Theta)$----------------((10)

according to theory of relativity $m= \frac{m_0}{\sqrt{(1-V^2/c^2}}$------------(11)

substitute eq (10) in eq(11)

$m_0^2c^2/(1-v^2/c^2)c^2(c^2-V^2)=m_0^2C^4+2hm_0c^2(\nu-\nu')- 2h^2\nu\nu'(1- \cos\Theta)$

$m_0^2/(c^2-v^2)\ c^4(c^2- v^2)= m_0^2C^4+2hm_0c^2(\nu -\nu')- 2h^2\nu\nu'(1- \cos\Theta)$

i.e., $2hm_0c^2(\nu -\nu')= 2h^2\ \nu\nu'(1- \cos\Theta)$

$(\nu-\nu')/ (\nu\nu') = h/ m_0c^2(1- \cos\Theta)$

$1/\nu -1/\nu' = \nu\ h/ m_0c^2(1- \cos\Theta)$

i.e $c(1/\nu-1/\nu') = \frac{h}{m_o c}(1- \cos\Theta)$

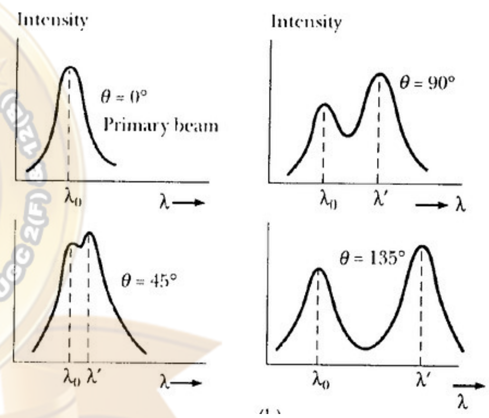
$\lambda^1- \lambda = \frac{h}{m_o c}(1- \cos\Theta)$

$\lambda= \lambda + \frac{h}{m_o c}(1- \cos\Theta)$ -------------------(11)

This is the relation between wavelength λ^1 of the scattring photon is grater than the wavelength λ of incident photon.

The change in wave length dλ(compton wave length) is given by

$d\lambda= \lambda^1- \lambda= \frac{h}{m_o c}(1- \cos\Theta)$ -----------(12)



we find that the change in wavelength dλ is independent of the wave length of the incident radiation λ as well as the nature of the scattring object.

It found to be depend on the angle of scattering , varying directly as $(1- \cos\Theta)$

i) When Θ =, 0 $\cos\Theta =1$, and hence $(1- \cos\Theta)=0$

i.e, $\lambda^1-\lambda=0$

i.e there is no scattring along the direction of incident.

ii) When $\Theta =\pi/2$, $\cos\Theta =0$ and hence $(1-\cos\Theta)=1$

i.e $\lambda^1- \lambda= \frac{h}{m_o c}$

iii) When $\Theta =\pi$, $\cos\Theta =-1$ and hence $(1- \cos\Theta)= 2$

i.e $\lambda^1- \lambda=\frac{2h}{m_o c}$

Hence as Θ varies from 0 to 180^0 , the wavelength of scattred radiation increases from λ to $(\lambda+2\ h /m_0c)$ provided the wavelength of the incident radiation is sufficently small,other wise there wil be no change in wavelength as predicated by the classical theory.

Matter waves

1.What are the matter waves (or)what is De-Broglie wavelength? (or) Explain the wave particle duality.

Ans.

- De-Broglie proposed the concept of matter waves, according to which a material particle of mass **'m'** moving with a velocity **'v'** should have an associated wavelength 'λ' called de-Broglie wavelength, and p its momentum.

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

- Wavelength is associated with moving particle and independent of charge of the particles.
- Greater the mass, velocity of the particle, lesser will be the wavelength.

2.Explain matter waves.

Ans.

- As electromagnetic waves behave like particles, particles like electrons will behave like waves called matter waves. The matter waves thus conceived by de Broglie are called de Broglie matter waves.
- When a particle of mass m travels with a velocity '**v**', de Broglie wave length of particle is given by $\boldsymbol{\lambda} = \frac{\mathbf{h}}{\mathbf{mv}} = \frac{\mathbf{h}}{\mathbf{p}}$where **h** is planks constant

Characteristics of the matter waves:

- According to de Broglie wavelength.

  1 If particle is lighter, then the wavelength of the matter waves is larger.

  2.The velocity is more, Wavelength is lesser

  3. If the particle is slower, Wavelength is more

  4. If v=0, (The particle is at rest), Wavelength is infinity. Hence matter waves are associated with moving material particle

  5. Whether the particle is charged or not, the matter waves are associated with it.

  6. The wave velocity of matter wave can be greater than the velocity of the light.

  7. No single phenomena exhibits both particle nature and wave nature simultaneously.

  8. The direct consequence of the wave nature of matter is Heisenberg uncertainty principle.

3.What is de-Broglie concept of dual nature of matter waves: -

Ans.

- In1924, Louisde-Brogliesuggestedthatmatterwavesalsoexhibitdualnaturelike radiation. They are

  i) Wave nature

  ii) Particle nature

- Wave nature of matter waves is verified by Davisson & Germer experiment, G.P.Thomson experiment etc.
- Particle nature of matter waves is verified by photo-electric effect, Compton effect etc.

4. Compare the properties of wave and particles

Ans.

Particle	Wave
1. A particle is a small quantity of matter under consideration	1. The wave is associated with to and from motion.
2. Localized point in space	2. Delocalized point in space
3. Energy depends upon mass and velocity of the particle $E = mc^2$	3. Energy depends upon wavelength and velocity $E=h\nu=(hc/\lambda)$
4. It does not show interference	4. It shows interference
5. Simultaneous existence of two or more particles	5. Simultaneous existence of two or more particles will be favored
6. It is characterized by mass, shape, size etc.,	6. It is characterized by frequency, amplitude etc.,

5. Write the Difference between matter waves and electro-magnetic waves.

Ans.

Matter Wave	Electromagnetic waves
1.The wave associated with moving material particle is known as matter waves.	1.Oscillating charged particle gives rise to EM wave.
2.Wavelengthofmatterwaveisgivenasλ=h/mv	2.Wavelengthofanelectromagneticwaveisgivenby λ=hc/E
3.Wavelength of matter wave depends up on mass of the particle & velocity.	3.Wavelengthofanelectromagnetic wave depends up on the energy of the photon.
4.Itcantravelwithavelocitygreaterthanthevelocity of light in vacuum.	4.Itcan travel with a velocity equal to the speed of light in vacuum i.e $c=3\times10^8$m/s.
5.It is not an EM wave.	5.Electric field and Magnetic field oscillate perpendicular to each other & generate EM Waves.

6.Explain the Heisenberg's Uncertainty principle.

Ans.

- From the de-Broglie hypothesis, we can confirm that the electron can behave both as a particle and as a wave.
- This dual behaviour of electron makes difficult in locating the exact position and momentum of the electrons, simultaneously. This is called uncertainty. difficulty is solved by Heisenberg in 1927, with the principle called as Heisenberg uncertainty principle.

Statement: It is impossible to measure both the position and momentum of a particle simultaneously to any desired degree of accuracy.

$$\Delta X.\Delta P \geq h/4\pi$$

Where ΔX is the uncertainty in the position and ΔP is the uncertainty in the momentum

$$\Delta E.\ \Delta t \geq h/4\pi$$

Where ΔE is the uncertainty in energy and Δt is the uncertainty in time

$$\Delta J.\ \Delta\theta \geq h/4\ \pi$$

Where ΔJ is the uncertainty in angular momentum and Δθ is the uncertainty in angular position.

Applications: -

1. It explains the non-existence of electrons in the nucleus.
2. It explains the existence of protons neutrons in the nucleus.
3. It gives the binding energy of an e-in atom.
4. It calculates the radius of Bohr's first orbit.

7. Explain wave-particle duality.

Ans.

- Light waves can behave like particles, i.e photons and waves
- This Phenomenon is called the wave-particle nature of light or wave-particle duality.
- Light interacts with matter, such as electrons, as a particle the evidence for this is provided by the photelectric effect
- Light propagates through space as a wave the evidence for this comes from the diffraction and interference of light in Young's Double slit experiment.

Light as a particle

- Einstein proposed that light can be described as a quanta of energy that behave as particles called photons.
- The photon model of light explains that:
  - Electromagnetic waves carry energy in discrete packets called photons
  - The energy of the photons is quantized according to the equation $E=h\nu$
  - In the photoelectric effect, each electron can absorb only a single photon – this means only the frequencies of light above the threshold frequency will emit a photoelectron

Wave-particle Duality

Einstein had shown that the momentum of a photon is

$$P = \frac{h}{\lambda} \quad \text{--------} \ 1$$

This can be easily shown as follows. Assuming $E = h\nu$ for a photon and $\lambda\nu = c$ for an electromagnetic wave, we obtain

$$E = \frac{hc}{\lambda} \quad \text{--------} \ 2$$

Now we use Einstein's relativity result $E = mc^2$ to find

$$\lambda = \frac{h}{mc} \quad \text{-------- } 3$$

which is equivalent to equation (1). Note that **m** refers to the relativistic mass, not the rest mass, since the rest mass of photon is zero. Since light can behave both as a wave (it can be diffracted, and it has a wavelength), and as particle (it contains packets of energy **h*ν***), de Broglie reasoned in 1924 that matter also can exhibit this wave particle duality. He further reasoned that matter would obey the same equation (1) as a light. In 1927, Davision and Germer observed diffraction patterns by bombarding metals with electrons, confirming de Broglie proposition.

8. Derive the wavelength of de Broglie wave.

Ans.

de-Broglie Hypothesis –Matter waves:

- The waves associated with moving material particles (like electrons, protons, and neutrons) are called matter waves or de-Broglie waves or piolet waves.
- The wave length of matter waves is derived based on radiation.

Equation of matter waves:

- According to Planck's theory of radiation, the energy of a photon is given by

$$E = h\nu = \frac{hc}{\lambda} \text{..... (1)}$$

where c is velocity of light , λ is wavelength of the photon, h= Planck's constant

- According to Einstein's mass energy relation,

$$E = mc^2 \text{--------- (2)}$$

Where m= mass of the photon

Equating equations (1) and (2),

$$mc^2 = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{mc^2} = \frac{h}{mc} = \frac{h}{p} \text{...... (3),}$$

where P = momentum of photon

This is known as de-Broglie general wave equation. It can be applicable to material particles as well as light waves.

- If m is mass of the material particle and v is the velocity, then the momentum associated with moving particle is p=mv.

From equation (3), we get

$$\lambda = \frac{h}{mv} = \frac{h}{p} \text{-------- (4)}$$

This is called de-Broglie's wave equation.

Other forms of de-Broglie wavelength(λ): -

In terms of Energy(E): -

If E is kinetic energy of the material particle, then

$$E=\frac{1}{2}mv^2 \text{ ---------------(5)}$$

Multiply Eq (5) by 'm' on both sides, we get

$$E=\frac{1}{2}mv^2 * \frac{m}{m}$$

$$2Em=m^2v^2$$

$$mv = \sqrt{2mE}$$

$$p = \sqrt{2mE}$$

$$\lambda = \frac{h}{mV} = \frac{h}{p}$$

hence de=Broglie wave length

$$\lambda = \frac{h}{\sqrt{2mE}} \text{ ---------------(6)}$$

ii) de-Broglie wavelength in terms of voltage(V)

➢ If a charged particle is accelerated through a potential difference(V), then the kinetic energy of the particle is given as

$$E=eV$$

But we have kinetic energy(E)of particle $=\frac{1}{2}mv^2$

$$eV = \frac{1}{2}mv^2$$

$$2eV = mv^2$$

Multiply by 'm' on both sides we get

$$2meV=m^2v^2$$

$$mv = \sqrt{2meV}$$

From de-Broglie general equation

$$\lambda = \frac{h}{mv} = \frac{h}{p}$$

$$\lambda = \frac{h}{\sqrt{2meV}} \quad -------- (7)$$

Where **h** is planks constant, **h**=6.625×10^{-34}J-S, m_o is rest mass of the electron $m_o=9.1\times10^{-31}Kg$ and e is the charge of electron $e=1.6\times10^{-19}c$

Substitute above values in eq (7), we get

$$\lambda = \frac{12.27}{\sqrt{V}} A^o$$

If the voltage applied to accelerate the electron is V = 100 volts then

$\lambda = \frac{12.27}{\sqrt{100}} A^0 = 1.226 A^0$

This expression is for *non-relativistic* case since relative variation of mass with velocity is not considered. Thus, accelerated electrons exhibit wave nature corresponding to X-Ray wavelength. This concept only helped Division and Germer to provide experimental evidence on matter waves when they conducted electron diffraction experiments.

Characteristics of Matter waves:

According to de Broglie wavelength

$$\lambda = \frac{h}{mv}$$

- Lighter the particle mass, greater is the wavelength associated with it.
- Lesser the velocity of the particle, longer is the wav-length associated with it.
- For V = 0, $\lambda = \infty$, This means that only moving particle is associated with matter wave.
- Whether the particle is charged or not, matter wave is associated with it. This means that these waves are not electromagnetic waves but a new kind of waves.
- It can be proved that the Matter waves travel faster than the velocity of light.

  We know that

  $$E = h\nu \text{ and } E = mc^2$$

  $$h\nu = mc^2$$

  $$\nu = \frac{mc^2}{h}$$

  The wave velocity (w) is given by

  $$\omega = \nu\lambda = \frac{mc^2}{h}\lambda$$

  Substituting for λ we get

  $$\omega = \frac{mc^2}{h}\frac{h}{mV}$$

  $$\boldsymbol{\omega = \frac{c^2}{V}}$$

  As the particle velocity (**v**) cannot exceed velocity of light (**c**),$\boldsymbol{\omega}$ is greater than velocity of light.
- No single phenomena exhibit both particle nature and wave nature simultaneously
- Thus, the wave nature of matter gives an uncertainty in the position of the particle.

9. Describe the Experimental Verification of matter waves using Davision and Germer experiment.

Ans:

- There are two methods to verify the dual nature of matter waves. They are

  1. Davisson and Germer's experiment

  2. G.P. Thomson's experiment

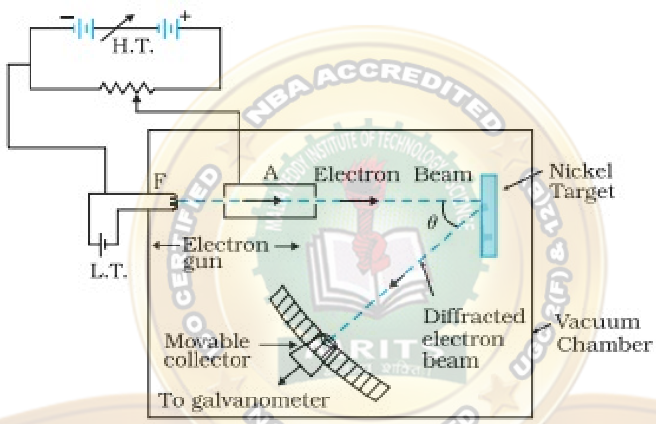
Davisson and Germer's experiment

- First practical evidence for the wave nature of matter waves was given by C.J.Davisson and L.H. Germer in 1927. This was the first experimental support to de-Broglie's hypothesis.

Principle:

- The electrons which are coming from the source are incident on the target and the electrons get diffracted. These diffracted electrons produce a diffraction pattern. It shows the wave nature of matter waves.

Experimental Arrangement:



Construction:

- The experimental setup is shown in above figure. It consists of mainly 3-parts

(i) Filament

(ii) Target

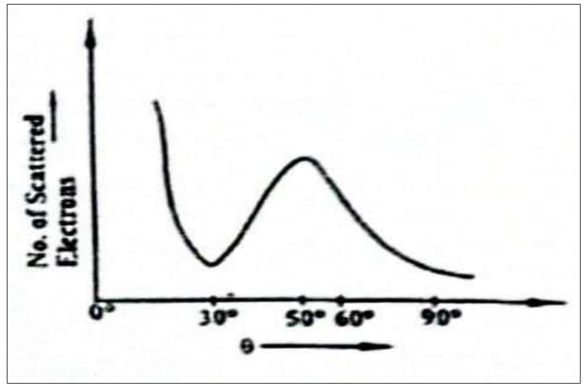
(iii) Circular scale arrangement.

It also consists of a low-tension battery, High tension battery & a cylinder(A).

Working:

- When tungsten filament 'F' is heated by a low-tension battery then electrons are produced. These electrons are accelerated by High tension battery. The accelerated electrons are collimated into a fine beam of pencil by passing them through a system of pin-holes in the cylinder 'A'.
- This beam of electrons can incident on nickel crystal which acts as target. Then electrons are scattered in all the directions. The intensity of scattered electrons is measured by the circular scale arrangement.
- In this arrangement, the movable collector is fixed to circular scale which can collect the electrons and can move along the circular scale. The electron collector is connected to a sensitive galvanometer to measure the intensity of electron beam entering the collector at different scattering angles (θ)

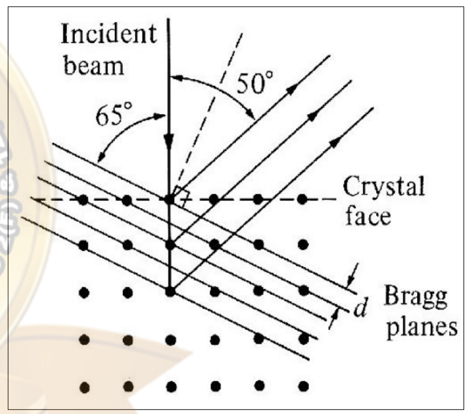
- A graph is plotted between the scattering angle (θ)and the Number of scattered electrons as shown in above figure(b).
- The intensity of scattered electron is maximum at θ = 50°& accelerating voltage = 54V.



Fig(b). The variation of the intensity of scattered electrons with the direction of scattering angle(θ)

Calculation of wave-length associated with electron's:

i) According to Bragg's equation



$2d\sin\theta = n\lambda$------------------- (1)

For nickel crystal, the inter planar distance **d** = 0. 909A° = 0.909×10^{-10}m

n is the order of diffraction, for first order n=1, and θ is the angle of diffraction.

From the figure,

$180^{\circ}=\theta+\theta+50^{\circ}$

$180^{\circ}=2\theta+ 50^{\circ}$

∴ Diffraction angle(θ)=65°----------(2)

Substituting the above values in equation (1) we get

$2\times0.909\times Sin65^{\circ}=1\times\lambda$

$\lambda=1.65A^{\circ}$----------------(3)

(ii)From de – Broglie wavelength(λ):

$$\lambda = \frac{12.27}{\sqrt{V}} A^0$$

But V= 54V

$$\lambda = \frac{12.27}{\sqrt{54}} A^0$$

$\lambda=1.67A^{\circ}$------------(4)

From equation (3) & equation (4) it was been proved both the practical & theoretical wavelengths are almost equal. Hence the wave nature of particle is proved experimentally.

10. Explain Physical significance of wave-function or Eigen−function (ψ).

Ans.

It is a variable or complex quantity that is associated with a moving particle at any position (x, y, z) and at any time 't'.

(i) 'ψ′ of a particle is represented by $\psi = \psi_0 e^{-i\omega t}$

(ii) 'ψ′ explains the motion of microscopic particles.

(iii) 'ψ′ is a complex quantity & it does not have any meaning.

(iv) $|\psi|2=\psi\psi*$ is real and positive, it has physical meaning.

(v) |ψ|2representstheprobabilityoffindingtheparticleperunitvolume.

(vi) For a given volume dτ, the probability of finding the particle is given by probability density(p)=$\iiint|\psi|2d\tau$ where dτ=dxdydz

(vii) 'ψ′ gives the information about the particle behaviour.

(viii) 'p'valuesarebetween0to1.

(xi) wave-function 'ψ′ is a single valued, finite and periodic function.

(x) If $p=\iiint|\psi|^2 d\tau=1$, then 'ψ′ is called normalized wave function.

11. Derive the Schrodinger's time independent wave equation for a particle?

Ans.

Schrodinger Wave Equation:

- Schrodinger describes the wave nature of a particle in mathematical form and is known as Schrodinger wave equation.

These are of 2 types:

1). Time independent wave equation. [$\nabla^2\psi + \frac{2m(E-V)}{h^2}\psi = 0$]

2). Time dependent wave equation. [E^Ψ= H^Ψ].

1. Schrodinger Time Independent Wave Equation

- Schrödinger, in 1926, developed wave equation for the moving particles. One of its forms can be derived by simply incorporating the de Broglie wavelength expression into the classical wave equation.
- If a particle of mass 'm' moving with velocity 'v' is associated with a group of waves.

Let ψbe the wave function of the particle. Also let us consider a simple form of progressing wave like the one represented by the following equation.

$$\Psi = \Psi_0 \sin(\omega t - k x) \text{ ------------------ } (1)$$

Where $\Psi = \Psi(x, t)$ and Ψ_0 is the amplitude

Differentiating Ψ partially w.r.to x,

$\frac{\partial \Psi}{\partial x} = \Psi_0 \cos(\omega t - k x)(-k) = -k\, \Psi_0 \cos(\omega t - kx)$

Once again differentiate w.r.to x,

$\frac{\partial^2 \Psi}{\partial x^2} = (-k)\, \Psi_0(-\sin(\omega t - k x))\,(-k)$

$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi_0 \sin(\omega t - kx)$

$\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \psi$ (from Eq (1))

$\frac{\partial^2 \Psi}{\partial x^2} + k^2 \psi = 0$ -----------(2)

$\frac{\partial^2 \Psi}{\partial x^2} + \left(\frac{4\pi^2}{\lambda^2}\right)\psi = 0$ ---------(3) (since $k = \frac{2\pi}{\lambda}$)

From eqn. (2) or eqn. (3) is the differential form of the classical wave eqn.

now we incorporate de-Broglie wavelength expression $\lambda = h/mv$.

Thus, we obtain, $\frac{\partial^2 \Psi}{\partial x^2} + \frac{4\pi^2}{(\frac{h}{mV})^2}\, \Psi = 0$

$\frac{\partial^2 \Psi}{\partial x^2} + \frac{4\pi^2 m^2 V^2}{h^2}\, \Psi = 0$--------------(4)

The total energy E of the particle is the sum of its kinetic energy K.E and potential energy V Total Energy=kinetic Energy+ Potential Energy

i.e., E = K.E + V -----------(5)

K.E = E-V

and $K.E = \frac{1}{2} m v^2 * \frac{m}{m} = \frac{m^2 v^2}{2m}$---------(6)

Therefore $m^2 v^2 = 2\, m\, (E - V)$ ------------(7)

Substitute equation (7) in (4), we get

$\frac{\partial^2 \Psi}{\partial x^2} + \left[\frac{8\pi^2 m}{h^2}(E-V)\right] \psi = 0$ ------------(8)

In quantum mechanics, $\hbar = h/2\pi$

$\frac{\partial^2 \Psi}{\partial x^2} + \left[\frac{2m}{\hbar^2}(E - V)\right]\psi = 0$ ------------(9)

For simplicity, we considered only one -dimensional wave.
Extending eqn. (9) for a three -dimensional, we get

$\frac{\partial^2 \Psi}{\partial x^2} + \frac{\partial^2 \Psi}{\partial y^2} + \frac{\partial^2 \Psi}{\partial z^2} + \left[\frac{2m}{\hbar^2}(E - V)\right]\psi = 0$ ------------(10)

$$\nabla^2 \psi + \frac{2m(E-V)}{\hbar^2}\psi = 0$$

This is known as **three-dimensional Schrodinger wave equation**.

$(where\ \nabla = i\frac{\partial}{\partial x} + j\frac{\partial}{\partial y} + k\frac{\partial}{\partial z}$ and $\nabla^2 = \frac{\partial^2\Psi}{\partial x^2} + \frac{\partial^2\Psi}{\partial y^2} + \frac{\partial^2\Psi}{\partial z^2}$)

12. Derive wave function and Eigen values of a particle enclosed in one dimensional potential box.

or

Derive an expression for the energy states of a Particle trapped in 1-Dimensional potential box.

Ans.

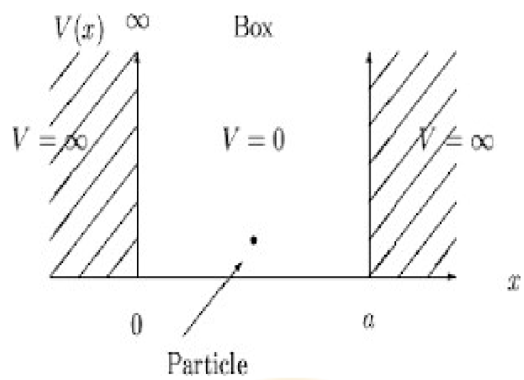


Fig: One dimensional potential well of infinite depth

- The wave nature of a moving particle leads to some remarkable consequences when the particle is restricted to a certain region of space instead of being able to move freely. i.e when a particle bounces back and forth between the walls of a box.
- The Schrodinger wave equation will be applied to study the motion of a particle in 1-D box to show how quantum numbers, discrete values of energy and zero-point energy arise.
- Consider a particle of mass 'm' moving freely along x- axis and is confined between x=0 and x=a by infinitely two hard walls, so that the particle has no chance of penetrating them and bouncing back and forth between the walls of a 1-D box.
- If the particle does not lose energy when it collides with such walls, then the total energy remains constant.

- This box can be represented by a potential well of width 'a', where V is uniform inside the box throughout the length 'a' i.e V= 0 inside the box or convenience and with potential walls of infinite height at x=0 and x=a, so that the PE 'V' of a particle is infinitely high V=∞ on both sides of the box.
- The position (x) of the particle at any instant is given by $0 < x < a$
- The potential energy of the particle is zero inside of the box and infinite outside of the box.

i.e., V=0 for $0 < x < a$

V= ∞ for $0 \geq x \geq a$

- Apply the Schrodinger's equation to describe the motion of the particle inside the box (V=0).

$\frac{\partial^2\Psi}{\partial x^2} + \frac{8\pi^2 m}{h^2}E\,\psi = 0$ --------------(1) as V=0 for a free particle

In the simplest form eq (1) can be written as

$\frac{\partial^2\Psi}{\partial x^2} + k^2 E\psi = 0$ ------------------- (2)

Where k = propagation constant and is given by $k = \sqrt{\frac{8\pi^2 mE}{h^2}}$

The general solution of this differential equation (2) is

$\psi(x) = A\sin kx + B\cos kx$ ------------(3)

Where A and B are arbitrary constants, and the value of these constant can be obtained by applying the boundary conditions.

The boundary condition is

$\psi(x) = 0$ at x = 0 -------------- (4)

$\psi(x) = 0$ at x = a--------------(5)

Substitute equation (4) in (3)

$0 = A\sin k(0) + B\cos k(0) \rightarrow$ B=0

$\psi(x) = A\sin kx$.............................. (6)

Substituting equation (5) in (6)

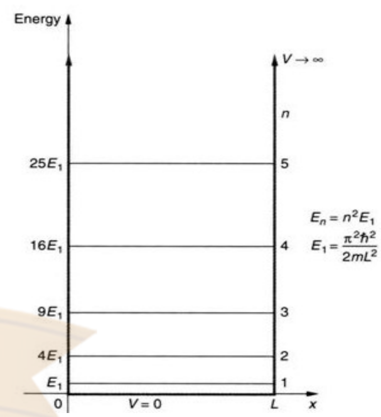
$0 = A\sin k(a)$

As A ≠0, $\sin ka = 0$

$ka = \sin^{-1}(0)$

$ka = n\pi$

$k = \frac{n\pi}{a}$...................... (7)



Where n= 1,2,3,4, and n≠0,because if n=0,k=0,E=0 everywhere inside the box and the moving particle cannot have zero energy.

From (8) $k^2 = \left(\frac{n\pi}{a}\right)^2$

From (5) $\frac{8\pi^2 mE}{h^2} = \frac{n^2\pi^2}{a^2}$

$$E = \frac{n^2h^2}{8ma^2}$$

$\boldsymbol{E_n = \frac{n^2h^2}{8ma^2}}$ **....................... (8)**

This equation gives the allowed values of energy for different values of n.

The lowest energy of a particle is given by putting n=1 in the equation (8)

$E_1 = \frac{h^2}{8ma^2}$ is the lowest energy, minimum energy, ground state energy or zero-point energy of the system.

$$E_n = n^2 E_1$$

The wave functions ψ_n corresponding to E_n are called Eigen functions of the particle, the integer 'n' corresponding to the energy E_n is called the quantum number of the energy level E_n.

Substituting (7) in (3)

$\psi_n = A sin\frac{n\pi x}{a}$................. (9)

- **Normalization of wave function**: The wave functions for the motion of the particle are

$$\psi_n = A sin\frac{n\pi x}{a}, for\ 0 < x < a$$

$$\psi_n = 0, for\ 0 \geq x \geq a$$

Calculation of A:

- According to normalization condition, the total probability that the particle is somewhere in the box must be unity.

$\int_0^a p_x dx = \int_0^a |\psi_n|^2 dx=1$

From equation (10)

$$\int_0^a A^2 sin^2\frac{n\pi x}{a} dx = 1$$

$$A^2 \int_0^a \frac{1}{2}\left[1 - \cos\frac{2\pi nx}{a}\right] dx = 1$$

$$\frac{A^2}{2}\left[x - \frac{a}{2\pi n} sin\frac{2\pi nx}{a}\right]_0^a = 1$$

The second term of the integrand expression becomes zero at both the limits.

$$\frac{A^2}{2}[a] = 1$$

$$A = \sqrt{\frac{2}{a}}$$

Therefore the wave function is $\psi_n = \sqrt{\frac{2}{a}}\ sin\frac{n\pi x}{a}$ ---------------- (10)

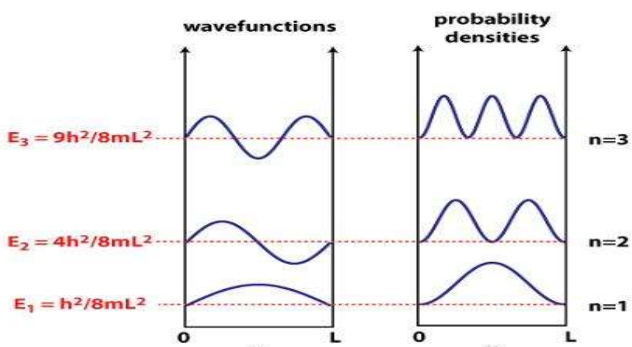


Fig. Energy level diagram corresponding to the normalized wave functions of the particle

Problems

1. Calculate the minimum wavelength of X-rays emitted when electrons accelerated through 30KV strike a target.

Sol.

$$\frac{1}{2}mv^2 = eV = hv = \frac{hc}{\lambda}$$

$$\lambda = \frac{hc}{eV} = \frac{6.626 \times 10^{-34} \times 3 \times 10^{8}}{1.6 \times 10^{-19} \times 30 \times 10^{3}} = 0.414 \times 10^{-10}m$$

= 0.414nm

2. In Compton Scattering the incident photons have wavelength 0.5nm. Calculate the wavelength of scattered radiation if they are viewed at an angle of 45° to the direction of the incidence.

Sol.

In Compton scattering we have

$$\lambda' - \lambda = \frac{h}{m_0 c}(1 - cos\theta)$$

$$\lambda = 0.5 \times 10^{-9}\, m; \qquad \theta = 45^{o}$$

$$\lambda' = 0.5 \times 10^{-9} + \left[\frac{6.626 \times 10^{-3} \times (1 - cos45^{\circ})}{9.1 \times 10^{-31} \times 3 \times 10^{8}}\right]$$

$\lambda' = (0.5 \times 10^{-9} + 0.00071 \times 10^{-9} = 0.5007\ nm.$

3. In a Compton Scattering the incident photons have a wavelength of 3 A° . What is the wavelength of the scattered photons if they are viewed at an angle of 60° to the direction of incidence?

Sol

$M_{e\ =\ 9.1\ X\ 10^{-31}}$ kg

h = 6.625 X 10^{-34} J

c= 3 X 10^{8} m/s

$$\lambda' - \lambda = \frac{h}{m_o c}(1 - cos\theta)$$

$$\lambda' = 3 \times 10^{-10} + \frac{6.626 \times 10^{-34}}{9.1 \times 10^{-31} \times 3 \times 10^{8}}(1 - cos60)$$

$$\lambda' = 3 \times 10^{-10} + 2.424 \times 10^{-12} \times 0.5 = 3.01212 \times 10^{-10}m$$

λ' =3.01212A^{0}

4. The wavelength of X-ray photon is doubled when it is scattered when it is scattered through an angle of 90° by a taget material. Find the incident wave length.

Sol.

$$\text{Formula } \lambda' - \lambda = \frac{h}{m_o c}(1 - cos\theta)$$

$$\text{Given } \lambda' = 2\lambda$$

$$\theta = 90^o$$

$$\text{Cos } 90 = 0$$

$$2\lambda - \lambda = \lambda = \frac{h}{m_o c}(1 - cos\theta)$$

$$= \frac{6.6\times10^{-34}\times1}{9\times10^{-31}\times3\times10^{8}} = 0.2444A^o$$

An electron is moving under a potential field of 15kV. Calculate the wavelength of the electron waves.

Sol: de-Broglie wavelength $\lambda = \frac{12.27}{\sqrt{V}} A^0$

$$\lambda = \frac{12.27}{\sqrt{1500}} A^0$$

$$\lambda = \frac{12.27}{122.47} A^0$$

$$\lambda = 0.1A^0$$

The wavelength of the electron waves = $0.1A^0$

5.Electrons are accelerated by 344 volts and are reflected from a crystal. The first reflection maximum occurs when the glancing angle is 60°. Determine the spacing of the crystal.

Sol: de-Broglie wavelength $\lambda = \frac{12.27}{\sqrt{V}} A^0$

$$\lambda = \frac{12.27}{\sqrt{344}} A^0$$

$$\lambda = 0.661A^0$$

According to Bragg's law

$$2d\sin\theta = n\lambda$$

For first reflection maximum n=1

$$\sin 60^o = 0.866$$

Inter atomic spacing of the crystal

$$d = \frac{n\lambda}{2\sin\theta}$$

$$d = \frac{0.661 \times 10^{-10}}{2 \times 0.866}$$

$$d = 0.3816 \times 10^{-10}\ m$$

$$d = 0.3816\ \text{A}^0$$

6. Calculate the wavelength associated with an electron with energy 2000 eV.

Sol: E = 2000 eV = 2000 × 1.6 × 10^{-19} J

$$\text{Kinetic energy } (E) = \frac{1}{2}mv^2 = \frac{p^2}{2m} \qquad \text{(or)} \qquad p = \sqrt{2mE}$$

$$\therefore\ \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}} = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 2000 \times 1.6 \times 10^{-19}}}\ \text{m}$$

$$= \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 1.6 \times 2000 \times 10^{-50}}} = \frac{6.63 \times 10^{-34}}{241.33 \times 10^{-25}}\ \text{m}$$

$$= 0.0275 \times 10^{-9}\ \text{m} = 0.0275\ \text{nm}$$

7. Calculate the velocity and kinetic energy of an electron of wavelength 1.66 × 10 $^{-10}$ m.

Sol: Wavelength of an electron (λ) = 1.66 × 10^{-10} m

$$\lambda = \frac{h}{mv}$$

$$v = \frac{h}{m\lambda} = \frac{6.63 \times 10^{-34}}{9.1 \times 10^{-31} \times 1.66 \times 10^{-10}}\ \text{m/s}$$

$$= \frac{6.63}{9.1 \times 1.66} \times 10^{7}\ \text{m/s} = 438.9 \times 10^{4}\ \text{m/s.}$$

To calculate KE:

$$\text{We know } E = \ P = \sqrt{2mE}$$

$$\text{and} \qquad \lambda = \frac{h}{p} = \frac{h}{\sqrt{2mE}}$$

$$\text{or} \qquad \lambda^2 = \frac{h^2}{2mE} \qquad \text{or} \qquad E = \frac{h^2}{2m\lambda^2} = \frac{\left[6.626 \times 10^{-34}\right]^2}{2 \times 9.1 \times 10^{-31} \times \left(1.66 \times 10^{-10}\right)^2}$$

$$= \frac{(6.626)^2 \times 10^{-68}}{2 \times 9.1 \times (1.66)^2 \times 10^{-51}}\ \text{J} = 8.754 \times 10^{-18}\ \text{J} = \frac{8.754 \times 10^{-18}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= 54.71\ \text{eV}$$

8. An electron is bound in one-dimensional infinite well of width 1 × 10^{-10} m. Find the energy values in the ground state and first two excited states.

Sol: Potential well of width (L) = 1 × 10^{-10} m

$$E_n = \frac{n^2 h^2}{8mL^2}$$

For ground state n = 1,

$$E_1 = \frac{h^2}{8mL^2} = \frac{\left[6.63\times10^{-34}\right]^2}{8\times9.1\times10^{-31}\times10^{-10}\times10^{-10}}\text{ J} = \frac{(6.63)^2}{8\times9.1}\times10^{-17}\text{ J}$$

$$= 0.6038\times10^{-17}\text{ J}$$

$$\text{(or)}\qquad = \frac{0.6038\times10^{-17}}{1.6\times10^{-19}}\text{ eV} = 37.737\text{ eV}$$

$E_2 = 4E_1 = 2.415 \times 10^{-17}$ J

$= 150.95$ eV

$E_3 = 9E_1 = 5.434 \times 10^{-17}$ J

$= 339.639$ eV.

9. An electron is bound in one-dimensional box of size 4 × 10^{-10} m. What will be its minimum energy?

Sol: Potential box of size (L) = 4 × 10^{-10} m

$$E_1 = \frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.1\times10^{-31}\times4\times10^{-10}\times4\times10^{-10}}\text{ J}$$

$$= \frac{6.63\times6.63}{8\times9.1\times16}\times10^{-17}\text{ J}$$

$$= 0.0377\times10^{-17}\text{ J}$$

$$\text{(or)}\qquad = \frac{0.0377\times10^{-17}}{1.6\times10^{-19}}\text{ eV}$$

$$= 2.3586\text{ eV}$$

10. An electron is moving under a potential field of 15 kV. Calculate the wavelength of the electron waves.

Sol: $V = 15 \times 10^3$ V $\quad \lambda = ?$

$$\lambda = \frac{1.227}{\sqrt{V}}\text{ nm} = \frac{1.227}{\sqrt{15000}}\text{ nm} = \frac{1.227}{122.47}\text{ nm} = 0.01\text{ nm}$$

11. Find the least energy of an electron moving in one-dimensional potential box (infinite height) of width 0.05nm.

Sol: $E_n = \dfrac{n^2h^2}{8mL^2} \qquad L = 0.05\text{ nm} = 0.05\times10^{-9}\text{ m}$

$$E_1 = \frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.1\times10^{-31}\times0.5\times10^{-10}\times0.5\times10^{-10}}\text{ J}$$

$$= \frac{6.63\times6.63}{8\times9.1\times0.25}\times10^{-17}\text{ J} = 2.4\times10^{-17}\text{ J}$$

$$= \frac{2.4\times10^{-17}}{1.6\times10^{-19}} = 150.95\text{ eV}$$

12. A quantum particle confined to one-dimensional box of width 'a' is known to be in its first excited state. Determine the probability of the particle in the central half.

Sol: Width of the box, L = a

First excited state means, n = 2

Probability at the centre of the well, P_2 (L/2) = ?

$$P_n(x) = \frac{2}{L}\sin^2\frac{n\pi x}{L}$$

$$P_2(L/2) = \frac{2}{L}\sin^2\frac{2\pi L/2}{L}$$

$$= \frac{2}{L}\sin^2\pi = 0$$

The probability of the particle at the centre of the box is zero.

13. An electron is confined in one-dimensional potential well of width 3×10^{-10} m. Find the kinetic energy of electron when it is in the ground state.

Sol: One-dimensional potential well of width, L = 3×10^{-10} m

Electron is present in ground state, so n = 1

E_1 =?

$$E_n = \frac{n^2h^2}{8mL^2}$$

$$E_1 = \frac{1^1 \times \left[6.63 \times 10^{-34}\right]^2}{8 \times 9.1 \times 10^{-31} \times \left[3 \times 10^{-10}\right]^2}\ \text{J} = 0.067 \times 10^{-17}\ \text{J}$$

or $$E_1 = \frac{0.067 \times 10^{-17}}{1.6 \times 10^{-19}}\ \text{eV}$$

$$= 4.2\ \text{eV}$$

14. Calculate the de Broglie wavelength of neutron whose kinetic energy is two times the rest mass of electron (given $m_n = 1.676 \times 10^{-27}$ kg, $m_e = 9.1 \times 10^{-31}$ kg, C = 3×10^{8} m/s and h = 6.63×10^{-34} J.S).

Sol: Kinetic energy of neutron, $\frac{1}{2}m_n v^2 = \frac{P_n^2}{2m_n} = 2m_e$

$P_n = \sqrt{4m_n m_e}$ where m_n = mass of neutron

m_e = mass of an electron

de Broglie wavelength of neutron, λ_n =?

$$\lambda_n = \frac{h}{P_n} = \frac{h}{\sqrt{4\,m_n m_e}} = \frac{6.63\times10^{-34}}{\sqrt{4\times9.1\times10^{-31}\times1.676\times10^{-27}}}$$

$$= \frac{6.63\times10^{-34}}{7.811\times10^{-29}}\text{ m} = 0.8488\times10^{-5}\text{ m} = 8488\text{ nm}.$$

15. An electron is confined to a one-dimensional potential box of length 2 Å. Calculate the energies corresponding to the second and fourth quantum states (in eV).

Sol: Length of the one-dimensional potential box, $L = 2\text{Å} = 2 \times 10^{-10}$ m

Energy of electron in nth level, $E_n = \frac{n^2h^2}{8mL^2} = n^2E_1$

$$\therefore\ E_1 = \frac{h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.1\times10^{-31}\times\left[2\times10^{-10}\right]^2}\text{ J}$$

$$= 0.150951 \times 10^{-17}\text{ J}$$

$$= \frac{0.150951\times10^{-17}}{1.6\times10^{-19}}\text{ eV} = 9.43\text{ eV}$$

Energy corresponding to second and fourth quantum states is:

$E_2 = 2^2E_1 = 4 \times 9.43$ eV $= 37.72$ eV

and

$E_4 = 4^2E_1 = 16 \times 9.43$ eV $= 150.88$ eV

16. Calculate the energy required to pump an electron from ground state to the 2nd excited state in a metal of length 10^{-10} m.

Sol: The energy of an electron of mass 'm' in nth quantum state in a metal of side 'L' is:

$$E_n = \frac{n^2h^2}{8mL^2} = n^2E_1$$

n = 1, corresponds to ground state

n = 2, corresponds to first excited state and

n = 3, corresponds to second excited state

$$E_1 = \frac{1^2h^2}{8mL^2} = \frac{\left(6.63\times10^{-34}\right)^2}{8\times9.11\times10^{-31}\times10^{-10}\times10^{-10}}\text{ J}$$

$$= 6.0314\times10^{-18}\text{ J} = \frac{6.0314\times10^{-18}}{1.6\times10^{-19}}\text{ eV} = 37.64\text{ eV}$$

$$E_3 = \frac{3^2h^2}{8mL^2} = 9E_1 = 9\times37.64\text{ eV} = 338.76\text{ eV}$$

17. Calculate the minimum energy of free electron trapped in a one-dimensional box of width 0.2 nm. (Given, h = 6.63 –10^{-34} J-S and electron mass × 9.1 × 10 $^{-31}$ kg)

Sol: One-dimensional box of width, L = 0.2 nm = 2 × 10^{-10} m

Minimum energy of the electron, E_1 =?

$$E_n = \frac{n^2 h^2}{8mL^2}$$

$$E_1 = \frac{1^2 \times \left[6.63 \times 10^{-34}\right]^2}{8 \times 9.1 \times 10^{-31} \times \left[2 \times 10^{-10}\right]^2} \text{ J} = 0.15095 \times 10^{-17} \text{ J}$$

$$= \frac{0.15095 \times 10^{-17}}{1.6 \times 10^{-19}} \text{ eV} = 9.43 \text{ eV.}$$

18. Calculate the wavelength associated with an electron raised to a potential 1600 V.

Sol: Potential (V) = 1600 V

Wavelength (λ) =?

$$\lambda = \frac{1.227}{\sqrt{V}} \text{ nm}$$

$$= \frac{1.227}{\sqrt{1600}} \text{ nm}$$

$$= 0.031 \text{ nm} = 0.31 \text{ Å.}$$

Objective Questions

1. The radiations emitted by hot bodies are called as ________________

a) X-rays **b) Black-body radiation**

c) Gamma radiations d) Visible light

2. A Black body

a) cannot emit light b) can emit only one wavelength

c) can emit all wavelengths d) can emit discrete wavelengths

3. During Einstein's Photoelectric Experiment, what changes are observed when the frequency of the incident radiation is increased.

a) The value of saturation current increases b) No effect

c) The value of stopping potential increases d) The value of stopping potential decreases

4. Compton shift depends on which of the following

a) Incident radiation b) Nature of scattering substance

c) Angle of scattering d) Amplitude of frequency

5. The wave associated with a moving particle is called

(a) longitudinal wave (b) mechanical wave

(c) matter wave (d) Planck's wave

6. Wave nature and particle nature called dual nature is exhibited by

a) particles only b) waves only

c) photons only **d) by both particles and waves**

7. de-Broglie Wavelength associated with a Particle of Mass 'm' moving with velocity 'v' is given by

a) λ=mv/h b) λ=hmv **c) λ=h/mv** d) λ=v/m

8. Consider an electron in the conduction band of a semiconductor with a kinetic energy of

0.2 eV. What is its de-Broglie wavelength?

a) 1.6 μm b) 1.6 nm c) 2.7 μm **d) 2.7 nm**

9. Existence of matter wave was experimentally first demonstrated by

a) Newton b) Planck

c) Davisson and Germer d) de Broglie

10. When an electron is accelrated by a potential 'V' volt, the de-Broglie wavelength is given by

a) 12.26/√v nm b) 26.12/√V Å c) 12.26/√ V μm **d) 12.26/√ V Å**

11. Which of the following is associated with an electron microscope

a) Matter waves b) Electrical waves

c) Magnetic waves d) Electromagnetic waves

12. The square of the magnitude of the wave function is called_________

a) current density **b) probability density**

c) zero density d) volume density

13. The total probability of finding the particle in space must be __________

(a) zero **(b) unity** (c) infinity (d) double

14. The wave function of the particle lies in which region

a) $x > 0$ b) $x < 0$ **c) $0 < X < L$** d) $x > L$

15. Wave nature and particle nature called dual nature is exhibited by

a) particles only b) photons only

c) waves only **d) by both particles and waves**

Fill in the blanks

1. The quantum of electromagnetic energy is called ______________.

2. The emission of electrons when a light of suitable wavelength falls on a metal plate is called______

3. According to Heisenberg's uncertainty principle, it is impossible to know both the exact position and exact ____________of an object at the same time.

4. If an electron is accelerated by a potential 100 volts, then its de-Broglie wavelength is____________

5. Lighter the particle, _____________ the de Broglie wavelength associated with it.

6. A potential difference appears across the depletion region and this potential is called____________ barrier potential

7. As intensity increases, the photoelectric effect ____________

8. The work function of lithium is 2.5eV. The maximum wavelength of light that can cause the photoelectric effect in lithium is ______________ nm

9. When a charged particle is accelerated through a potential difference V, its kinetic energy

10. In the Photoelectric effect, electrons should be removed from the ____________ As the temperature of a semiconductor increases its conductivity.

Questions

1. Derive the equation of Planck's radiation formula.

2. What are essential physical assumptions needed to explain the characteristics of Photoelectric effect.

3. Explain and derive the equation of Compton effect.

4. State and explain the Heisenberg's uncertainty principle.

5. Prove de Broglie's hypothesis using Division and Germer's experiment.

6. Derive an expression for time independent Schrodinger's wave equation.

7. Explain the Born interpretation of wave function.

8. Explain wave - particle duality.